A6.Anexă: Integrale pentru Soluții Cuantice

Integrala radială Slater

- $\boxed{I_k(m)=\int_0^{\infty} r^k e^{-mr}dr=\frac{k!}{m^{k+1}}}, \forall k\in N \,\&\, m\in C, \mathrm{Re}(m)>0$ ...radială Slater

Rezolvare:

$$I_k(m)=\int_0^{\infty}\frac{d^k\left(e^{-mr}\right)}{d(-m)^k}dr=(-1)^k\frac{d^k}{dm^k}\int_0^{\infty}e^{-mr}dr=(-1)^k\frac{d^k}{dm^k}\left[-\frac{1}{m}e^{-mr}\right]_0^{\infty}$$

$$=(-1)^k\frac{d^k}{dm^k}\left(m^{-1}\right)=(-1)^k(-1)^k k!m^{-1-k}=\frac{k!}{m^{k+1}}$$

Integrale de tip Poisson

- $\boxed{I_0(a)=\int_{-\infty}^{+\infty}e^{-ax^2}dx=\sqrt{\frac{\pi}{a}}}$… integrala de ordinul 0 de tip Poisson

Rezolvare:

$$I_0^2(a)=\left(\int_{-\infty}^{+\infty}e^{-ax^2}dx\right)\left(\int_{-\infty}^{+\infty}e^{-ay^2}dy\right)=\iint_{-\infty}^{+\infty}e^{-a\left(x^2+y^2\right)}dxdy$$

$$=\int_0^{\infty}\int_0^{2\pi}e^{-ar^2}rdrd\varphi=\left(\int_0^{\infty}e^{-ar^2}rdr\right)\left(\int_0^{2\pi}d\varphi\right)=-\frac{2\pi}{2a}\int_0^{\infty}d\left(e^{-ar^2}\right)=-\frac{\pi}{a}\left(e^{-ar^2}\right)_0^{\infty}=\frac{\pi}{a}$$

- $\boxed{I_1(a)=\int_{-\infty}^{+\infty}xe^{-ax^2}dx=0}$ … integrala de ordinul 1 de tip Poisson

Rezolvare:

$$I_1(a)=\int_{-\infty}^{+\infty}xe^{-ax^2}dx=-\frac{1}{2a}\int_{-\infty}^{+\infty}d\left(e^{-ax^2}\right)=-\frac{1}{2a}\left(e^{-ax^2}\right)_{-\infty}^{+\infty}=0$$

- $\boxed{I_2(a)=\int_{-\infty}^{+\infty}x^2e^{-ax^2}dx=\frac{1}{2a}\sqrt{\frac{\pi}{a}}}$ … integrala de ordinul 2 de tip Poisson

Rezolvare:

$$I_2(a)=\int_{-\infty}^{+\infty}x^2e^{-ax^2}dx=\int_{-\infty}^{+\infty}x\left(xe^{-ax^2}\right)dx=-\frac{1}{2a}\int_{-\infty}^{+\infty}x\frac{d}{dx}\left(e^{-ax^2}\right)dx$$

$$=-\frac{1}{2a}\left[\int_{-\infty}^{+\infty}\frac{d}{dx}\left(xe^{-ax^2}\right)dx-\int_{-\infty}^{+\infty}e^{-ax^2}dx\right]=-\frac{1}{2a}\underbrace{\left(xe^{-ax^2}\right)_{-\infty}^{+\infty}}_{(l'Hospital)\to 0}+\frac{1}{2a}\sqrt{\frac{\pi}{a}}=\frac{1}{2a}\sqrt{\frac{\pi}{a}}$$